# Adaptive management of the loyalty program by a game theory approach

Cite as: AIP Conference Proceedings **2172**, 080010 (2019); https://doi.org/10.1063/1.5133568
Published Online: 13 November 2019

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# Adaptive Management of the Loyalty Program by a Game Theory Approach

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**Abstract.** In this paper, we consider modern loyalty programs that use CRM (Customer Relationship Management). Ordinary discount cards do not allow to fully manage repeated touches with customers. The main task of the seller is to find ways to keep customers coming back. There are encouraged to use mathematical modeling using the game theory. This will allow the seller and the buyer to find the best strategies. In addition, the possibility of finding the best strategies of the seller in case of a change in the matrix of profits due to certain circumstances is considered. In this case, the behavior of the customers flow is given by a highly likely mixed strategy.

## INTRODUCTION

The selection strategies of the seller in the loyalty program is considered. The goal is to increase the profit of the seller. This can be achieved by retaining your customers and attracting new customers. Members of the loyalty program may have special privileges, the so–called VIP status with more favorable conditions of purchase. This encourages them to make even more purchases. Special conditions for VIP customers are determined by the seller. As for attracting new customers, here we can talk about privileges for potential participants. These are starting bonuses, special conditions for the first or second purchase. In general, the seller needs to manage his loyalty program as much as possible.

From the point of view of the mathematical theory of games, the seller has many strategies to increase their profits if the seller has ideas about the possible behaviors of the buyer. This situation can be modeled using the classic single player game under uncertainty. On the other hand, it will be interesting to consider the optimal behavior of the buyer. The buyer, in turn, has several strategies that will ensure the profitability of his purchase. This is especially true today developing online commerce. A buyer making a successful purchase on the Internet assesses the merit of such a service and, as a rule, has a motive to make repeated purchases.

First there were discounts, customer club cards came to replace them, then the opportunity to create a personal account of the buyer appeared. Today there are common platforms that connect the buyer and the seller. Modern loyalty programs are integrated with social networks. They also contain a data analysis package, a statistics module. The decision maker receives data in real time and has the ability to build an adaptive strategy.

There is another aspect of the application of the game approach in managing a loyalty program. Gamification is the application of game-design elements and game principles in non-game contexts. Gamification is an exciting trend that is quickly becoming the focus of many conversations about rewards and retention marketing. Gamification seems to be everywhere, with an increasing number of rewards programs incorporating it into their structure and reaping the benefits it offers. Gamification is not about building something new, but rather amplifying something that already works by applying data-driven engagement techniques to motivate users and create value. When we think about it this way, the mechanics that drive gamification sound an awful lot like the same ones that enhance VIP reward tiers. Leveraging customer motivations like status and achievement, VIP tiers are a proven way to boost program

*Proceedings of the 45th International Conference on Application of Mathematics in Engineering and Economics (AMEE'19)*
AIP Conf. Proc. 2172, 080010-1–080010-7; https://doi.org/10.1063/1.5133568
Published by AIP Publishing. 978-0-7354-1919-3/$30.00

engagement and increase success. As a result, they are the perfect vehicle for incorporating gamification into your business model.

Keeping customers loyal is marketing, and yet companies often spend much less on customer retention than customer acquisition. Forrester research found in 2016 that B2C marketers devoted 37% of their budget to attracting new customers and only 20% of their budget to retaining existing clients. This is despite research that shows that customer retention can pay off between 25% and 125% annually, with existing customers 60–70% more likely to convert than new customers. It is easier to retain than to find new, but companies are often focused on getting that next big customer rather than growing their existing relationships.

Loyalty programs can be found everywhere from coffee shop to mobile applications. Usually based on a point or badge system, customer loyalty gamification rewards customers for purchasing and sharing their experience with others. Common rewards include discounts and free gifts, free shipping, or other valuable prizes for engagement. By targeting specific behaviors and offering relevant reward systems, gamification provides an excellent method for rewarding your most loyal customers.

The application of the principles of Game Theory to the formulation of business and marketing strategy would be worthwhile. Neumann and Morgenstern in book [3] identified two types of games. The first is rule-based games where players interact according to specified rules of engagement. In the second type, freewheeling games, players interact without any external constraints. Business is a complex of both types of games.

## PROBLEM STATEMENT

A number of indicators are used to predict the needs of potential buyers. This may be the level of consumption before and after registering with the program, i.e. changes in customer activity after entering the program, the outflow rate among participants and non-program participants, as a loyalty program really affects customer retention, comparing incomes of participants and non-program participants in terms of the difference in customer value between members and non-program members, participation rate i.e. the ratio of the number of program participants and the total number of clients, the added value of the program, i.e. a measure of the added value of a company loyalty program.

The calculation of the above effects and indicators takes a lot of time and money, and therefore requires an automated calculation. Statistics and analytics tools allow you to get information about the history of purchases to promote more personalized stocks and evaluate the effectiveness of all services for a control period of time. Thus, an effective system of interaction between users of customers and entrepreneurs is built. Moreover, both parties can make certain decisions similar to the choice of optimal strategies in game theory [1].

Using the methods of operations research [4] – [7] and, in particular, the classical game theory, will undoubtedly increase the effectiveness of decisions made. For entrepreneurs, it is desirable to observe a positive trend in the growth of new customers. And for customers it is advisable to choose the optimal strategy for disposing of points and bonuses in terms of increasing their discounts. As a result, this interaction situation can be described as a matrix game of dimension $m \times n$, where $m$ is the number of entrepreneur strategies, $n$ is the number of client strategies, and use the data on the effectiveness of the adopted strategy or mixed strategies from the point of view of the first player as elements of the payoff matrix

$$P = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}. \tag{1}$$

Using summarized case data from the Russian company Global Intellect Service, we can write out a Table 1 of

**TABLE 1.** Net profit of an entrepreneur, RUB

| | **Spend bonuses** | **Save bonuses** | **Recommend** | **Earn bonuses out** | **Not active** |
|---|---|---|---|---|---|
| **VIP Promotion** | 33000 | 25000 | 36000 | 47000 | 26000 |
| **Customer retention** | 28000 | 21000 | 30000 | 48000 | 29000 |
| **Sale to customers** | 34000 | 35000 | 29000 | 27000 | 31000 |
| **Brand attractiveness** | 26000 | 27000 | 32000 | 28000 | 27000 |

probable profits depending on the implementations of the various strategies of the game participants. Now we can find the optimal strategies of the first and second players for a given payoff matrix.

## CALCULATION OF OPTIMAL MIXED–STRATEGIES

Let's write the resulting game problem as a linear programming problem [2]. In order to find the optimal entrepreneurial strategies and the optimal customer's strategies, it is necessary to reduce the game problem to dual linear programming problems. Let the game be set by the payoff matrix

$$P = \begin{pmatrix} 33000 & 25000 & 36000 & 47000 & 26000 \\ 28000 & 21000 & 30000 & 48000 & 29000 \\ 34000 & 35000 & 29000 & 27000 & 31000 \\ 26000 & 27000 & 32000 & 28000 & 27000 \end{pmatrix}. \tag{2}$$

Denote $p_1, p_2, p_3, p_4$ shares of the first player's optimal mixed–strategy. Let

$$x_1 = p_1/\nu,\ x_2 = p_2/\nu,\ x_3 = p_3/\nu,\ x_4 = p_4/\nu, \tag{3}$$

where $\nu$ is payoff. Then the corresponding linear programming problem can be formulated as follows

$$x_1 + x_2 + x_3 + x_4 \to \min$$

$$\begin{cases} 33000x_1 + 28000x_2 + 34000x_3 + 26000x_4 \geq 1 \\ 25000x_1 + 21000x_2 + 35000x_3 + 27000x_4 \geq 1 \\ 36000x_1 + 30000x_2 + 29000x_3 + 32000x_4 \geq 1 \\ 47000x_1 + 48000x_2 + 27000x_3 + 28000x_4 \geq 1 \\ 26000x_1 + 29000x_2 + 31000x_3 + 27000x_4 \geq 1 \end{cases}$$

Here $x_i \geq 0$, $i = 1, \ldots, 4$. The calculation results are shown in Figure 1.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | 33000 | 28000 | 34000 | 36000 | 0 | 1,11011 | |
| 3 | | 25000 | 21000 | 35000 | 29000 | 0 | 1,0801 | |
| 4 | | 36000 | 30000 | 29000 | 33000 | 0,36404 | 1 | |
| 5 | | 38000 | 48000 | 27000 | 28000 | 0,03947 | 1 | |
| 6 | | 26000 | 29000 | 31000 | 27000 | 0,59649 | 1 | |
| 7 | | 30114 | 30114 | 30114 | 29223,7 | | | |
| 8 | | 5E-06 | 2,3E-06 | 2,6E-05 | 0 | | | |
| 9 | | | | | | | | |
| 10 | | 3,3E-05 | | | | | | |
| 11 | | | | | | | | |
| 12 | v = | 30114 | | | | | | |
| 13 | | | | | | | | |
| 14 | Pi = | 0,14912 | 0,07018 | 0,7807 | 0 | | 1 | |

**FIGURE 1.** Calculating the optimal mixed–strategy of entrepreneur

Denote $q_1, q_2, q_3, q_4, q_5$ shares of the second player's optimal mixed–strategy. Let

$$y_1 = q_1/\nu,\ y_2 = q_2/\nu,\ y_3 = q_3/\nu,\ y_4 = q_4/\nu,\ y_5 = q_5/\nu. \tag{4}$$

Then the corresponding linear programming problem can be formulated as follows

$$y_1 + y_2 + y_3 + y_4 + y_5 \to \max$$

$$\begin{cases} 33000y_1 + 25000y_2 + 36000y_3 + 47000y_4 + 26000y_5 \le 1 \\ 28000y_1 + 21000y_2 + 30000y_3 + 48000y_4 + 29000y_5 \le 1 \\ 34000y_1 + 35000y_2 + 29000y_3 + 27000y_4 + 31000y_5 \le 1 \\ 26000y_1 + 27000y_2 + 32000y_3 + 28000y_4 + 27000y_5 \le 1 \end{cases}.$$

Here $y_i \ge 0$, $i = 1, \ldots, 5$. The calculation results are shown in Figure 2.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | 33000 | 25000 | 36000 | 38000 | 26000 | | 1 |
| 3 | | 28000 | 21000 | 30000 | 48000 | 29000 | | 1 |
| 4 | | 34000 | 35000 | 29000 | 27000 | 31000 | | 1 |
| 5 | | 36000 | 29000 | 33000 | 28000 | 27000 | | 0,97043 |
| 6 | | | | | | | | |
| 7 | | 0 | 0 | 1,2E-05 | 1,3E-06 | 2E-05 | | |
| 8 | | | | | | | | |
| 9 | | 3,3E-05 | | | | | | |
| 10 | | | | | | | | |
| 11 | v = | 30114 | | | | | | |
| 12 | | | | | | | | |
| 13 | qj = | 0 | 0 | 0,36404 | 0,03947 | 0,59649 | | 1 |

**FIGURE 2.** Calculating the optimal mixed–strategy of customer

So the optimal mixed–strategies of both players are obtained. For the entrepreneur (seller), the best strategy consists of three parts. These are "VIP Promotion" (14.9%), "Customer retention" (7%) and "Sale to customers"(78.1%). For the customer (buyer) the best strategy consists of three parts. These are "Recommendations" (36.4%), "Earn bonuses out" (3.9%) and "Inactivity" (59.7%). These values can be seen in figures 1 and 2 in the green field of the last line. The optimal payoff value can be seen in the yellow field. It is equal to $\nu = 30114$RUB.

## ADAPTIVE MANAGEMENT

The possibility of finding the best strategies of the seller in case of a change in the matrix of profits due to certain circumstances is of interest. In this case, the behavior of the customers flow is given by a highly likely mixed strategy. Because the assumption that each buyer will act in accordance with the optimal strategy is unlikely. And it will be the most pessimistic profit forecast of the seller. Here some generalized aligned mixed customer flow strategy can be considered. For example, $q = (0.2; 0.2; 0.2; 0.2; 0.2)$ and by each i-th position can be considered deviations $q_i = 0.2 \pm 0.1$ subject to $\Sigma q_i = 1$. With this rule, 21 variants of the buyers mixed strategies are possible (see Table 2).

**TABLE 2.** Variants of the buyers strategies

| | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ |
|---|---|---|---|---|---|
| **1** | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| **2** | 0.1 | 0.3 | 0.2 | 0.2 | 0.2 |
| **3** | 0.1 | 0.2 | 0.3 | 0.2 | 0.2 |
| **4** | 0.1 | 0.2 | 0.2 | 0.3 | 0.2 |
| **5** | 0.1 | 0.2 | 0.2 | 0.2 | 0.3 |
| **6** | 0.3 | 0.1 | 0.2 | 0.2 | 0.2 |
| **7** | 0.2 | 0.1 | 0.3 | 0.2 | 0.2 |
| **8** | 0.2 | 0.1 | 0.2 | 0.3 | 0.2 |
| **9** | 0.2 | 0.1 | 0.2 | 0.2 | 0.3 |
| **10** | 0.3 | 0.2 | 0.1 | 0.2 | 0.2 |
| **11** | 0.2 | 0.3 | 0.1 | 0.2 | 0.2 |
| **12** | 0.2 | 0.2 | 0.1 | 0.3 | 0.2 |
| **13** | 0.2 | 0.2 | 0.1 | 0.2 | 0.3 |
| **14** | 0.3 | 0.2 | 0.2 | 0.1 | 0.2 |
| **15** | 0.2 | 0.3 | 0.2 | 0.1 | 0.2 |
| **16** | 0.2 | 0.2 | 0.3 | 0.1 | 0.2 |
| **17** | 0.2 | 0.2 | 0.2 | 0.1 | 0.3 |
| **18** | 0.3 | 0.2 | 0.2 | 0.2 | 0.1 |
| **19** | 0.2 | 0.3 | 0.2 | 0.2 | 0.1 |
| **20** | 0.2 | 0.2 | 0.3 | 0.2 | 0.1 |
| **21** | 0.2 | 0.2 | 0.2 | 0.3 | 0.1 |

Then a Bayesian game can be used, in which players have incomplete information about other players. In addition to the actual players in the game, there is a special player called Nature (in the case considered here it is the buyer). The

lack of information held by players and modeling of beliefs mean that such games are also used to analyse imperfect information scenarios.

Mean values of the seller's profits when implementing possible buyer's strategies from Table 2 is shown in Figure 3 using the diagram. Now we can choose the highest profit in each of the strategy groups $S_1$, $S_2$, $S_3$, $S_4$. For

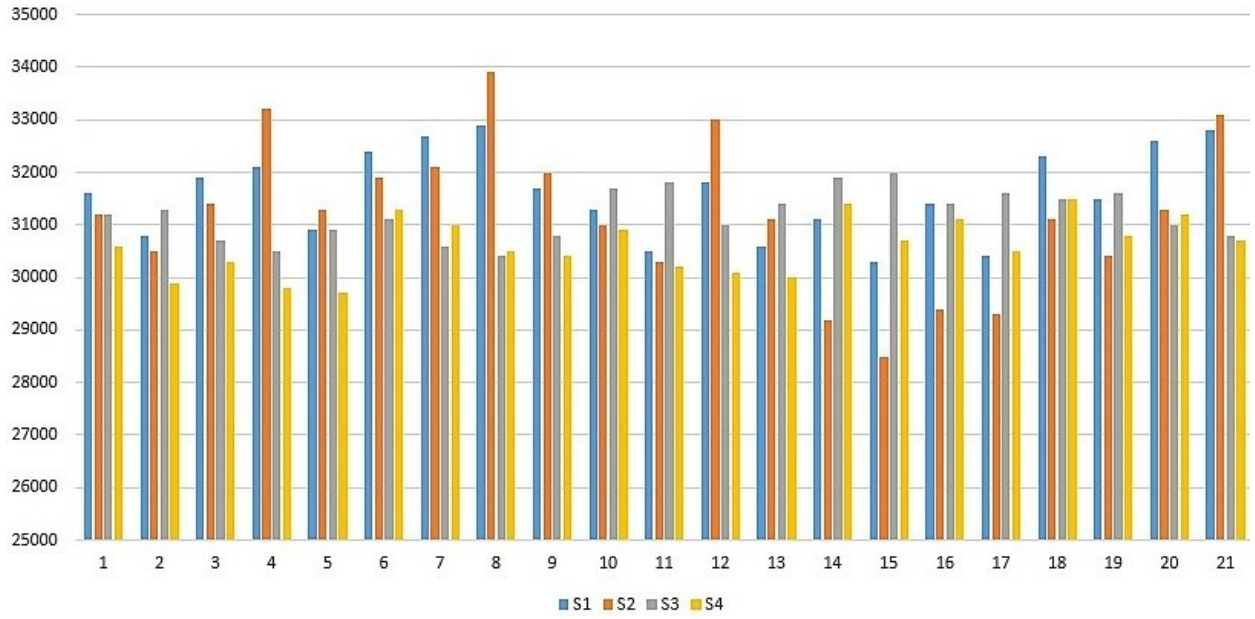


**FIGURE 3.** Mean values of the seller's profits

example, in the first group the strategy $S_1$ is the best, in the second group the strategy $S_3$ is the best, in the third group the strategy $S_1$ is the best and so on.

Let's consider the case when, for some reason, the profit from the implementation of the third strategy of the seller (Sale to customers) decreased by 10000RUB. What consequences will arise and what adaptive management in the loyalty program should the seller undertake?

Using the approach from previous section, one can find the optimal mixed strategy for the seller to ensure maximum profit. Moreover, this maximum profit corresponds to the pessimistic forecast, since the buyer, in turn, uses his optimal strategy. The calculation results for new payoff matrix are shown in Figures 4 and 5.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | 33000 | 28000 | 24000 | 36000 | 0 | 1,25547 | |
| 3 | | 25000 | 21000 | 25000 | 29000 | 0,2 | 1 | |
| 4 | | 36000 | 30000 | 19000 | 33000 | 0 | 1,18248 | |
| 5 | | 38000 | 48000 | 17000 | 28000 | 0 | 1,16788 | |
| 6 | | 26000 | 29000 | 21000 | 27000 | 0,8 | 1 | |
| 7 | | 25800 | 27400 | 21800 | 27400 | | | |
| 8 | | 0 | 7,3E-06 | 0 | 2,9E-05 | | | |
| 9 | | | | | | | | |
| 10 | | 3,6E-05 | | | | | | |
| 11 | | | | | | | | |
| 12 | v = | 27400 | | | | | | |
| 13 | | | | | | | | |
| 14 | Pi = | 0 | 0,2 | 0 | 0,8 | | 1 | |

**FIGURE 4.** Calculating the optimal mixed–strategy of entrepreneur

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | |
| 2 | | 33000 | 25000 | 36000 | 38000 | 26000 | | 0,94161 |
| 3 | | 28000 | 21000 | 30000 | 48000 | 29000 | | 1 |
| 4 | | 24000 | 25000 | 19000 | 17000 | 21000 | | 0,79562 |
| 5 | | 36000 | 29000 | 33000 | 28000 | 27000 | | 1 |
| 6 | | | | | | | | |
| 7 | | 0 | 7,3E-06 | 0 | 0 | 2,9E-05 | | |
| 8 | | | | | | | | |
| 9 | | 3,6E-05 | | | | | | |
| 10 | | | | | | | | |
| 11 | v = | 27400 | | | | | | |
| 12 | | | | | | | | |
| 13 | qj = | 0 | 0,2 | 0 | 0 | 0,8 | | 1 |

**FIGURE 5.** Calculating the optimal mixed–strategy of customer

The optimal mixed–strategies of both players are as follows. For the entrepreneur (seller), the best strategy consists of two parts. These are ”Customer retention” (20%) and ”Brand attractiveness”(80%). For the customer (buyer) the best strategy consists of two parts too. These are ”Save bonuses” (20%) and ”Inactivity” (80%). These values can be seen in Figures 4 and 5 in the green field of the last line. The optimal payoff value can be seen in the yellow field. It is equal to $v = 27400$RUB. Thus, a significant drop in profits in one of the strategies does not lead to catastrophic consequences in the expected profits from the entire loyalty program. The decrease was only 10%.

Let’s consider a generalized customer flow strategy in accordance with the rule given in Table 2. Using the Bayesian game in which players have incomplete information about other players, we find the average values of the sellers profit when implementing possible buyer’s strategies from Table 2. The results are shown in Figure 6 using a chart.

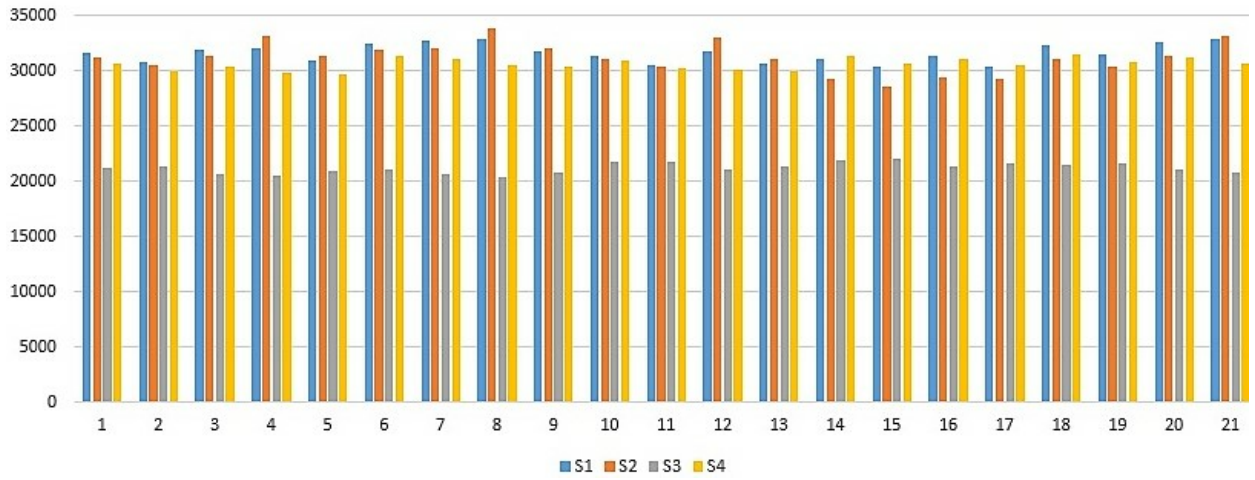


**FIGURE 6.** Mean values of the seller’s profits

Here we can choose the highest profit in each of the strategy groups $S_1$, $S_2$, $S_3$, $S_4$. Hence, in the first, second, third groups the strategies $S_1$ are the best, in the fourth group the strategy $S_2$ is the best and so on.

## CONCLUSION

Customer analytics play a key role in helping organizations optimize and support key junctures throughout the entire customer journey where the goal is to ensure that whatever type of interaction or channel used, the customer receives a consistent, personalized and compelling experience. When one customer experience after another is positive, loyalty is follow.

Important aspects and sources for the decision maker are deepen knowledge of customers preferences, understanding social relationships between your customers and discover how those relationships influence purchase behavior and loyalty, deliver anticipated, relevant offers and making more accurate predictions about customers future buying behaviors.

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